Steel structures, selected chapters

Part 4, Fatigue: service life, calculations by fracture mechanics

(ref. TGC 10 section 13.4)

RESSLab
Resilient Steel Structures Laboratory

Content

- Propagation law (Paris law)
- Lifetime Calculation
- Parameters influencing the service life
- Thickness effect in welded joints
  $(\Delta\sigma_{C,\boldsymbol{red}} = \boldsymbol{k_s} \cdot \Delta\sigma_C)$

Fig. 13.9: Stages of crack development

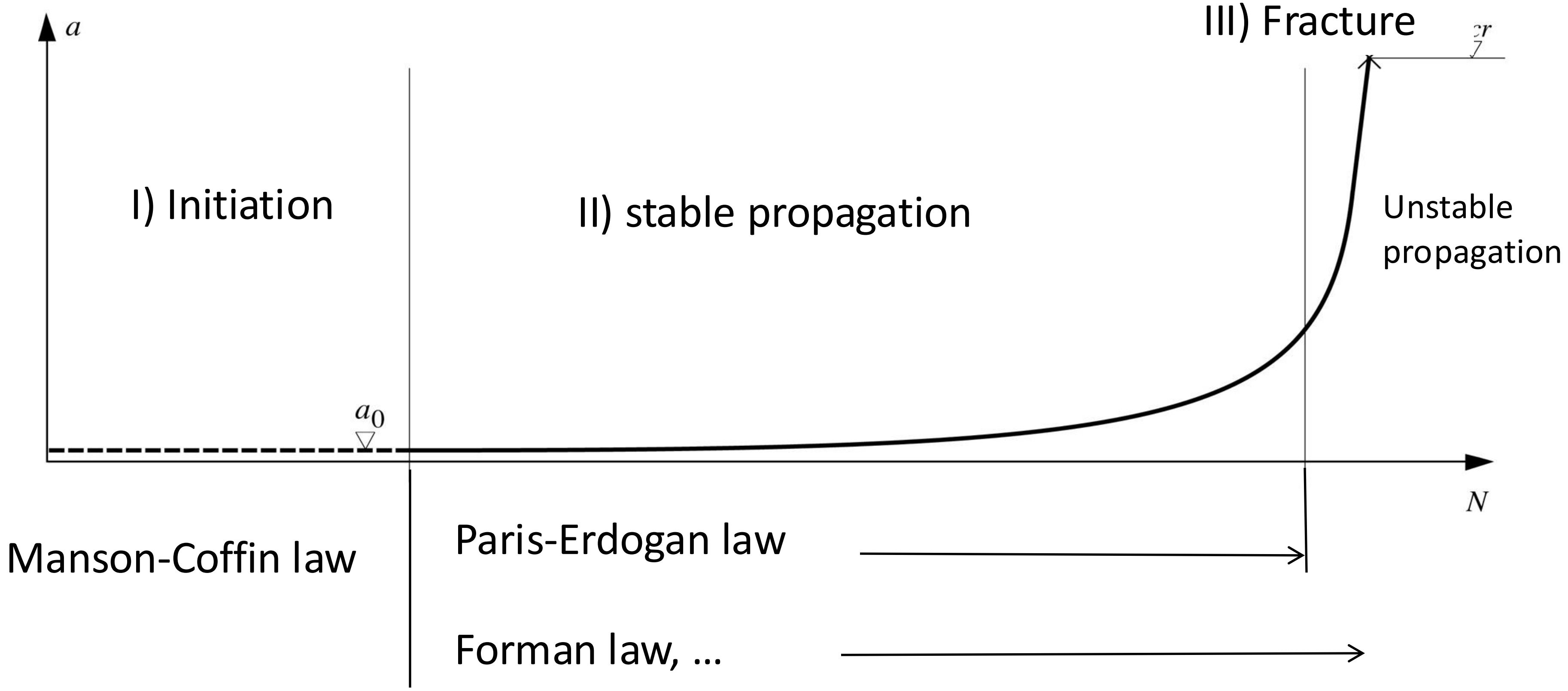


Fig. 13.9: Stages of crack development

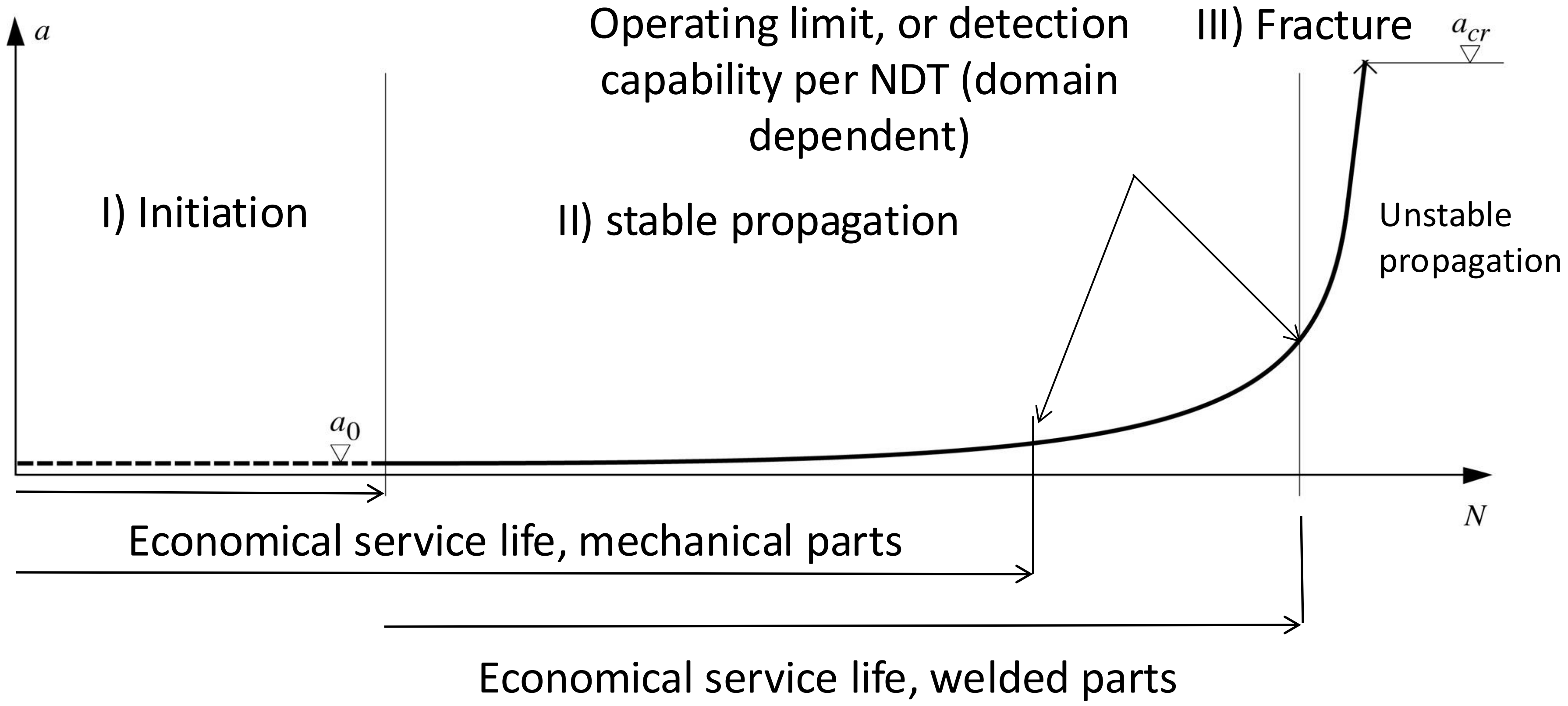
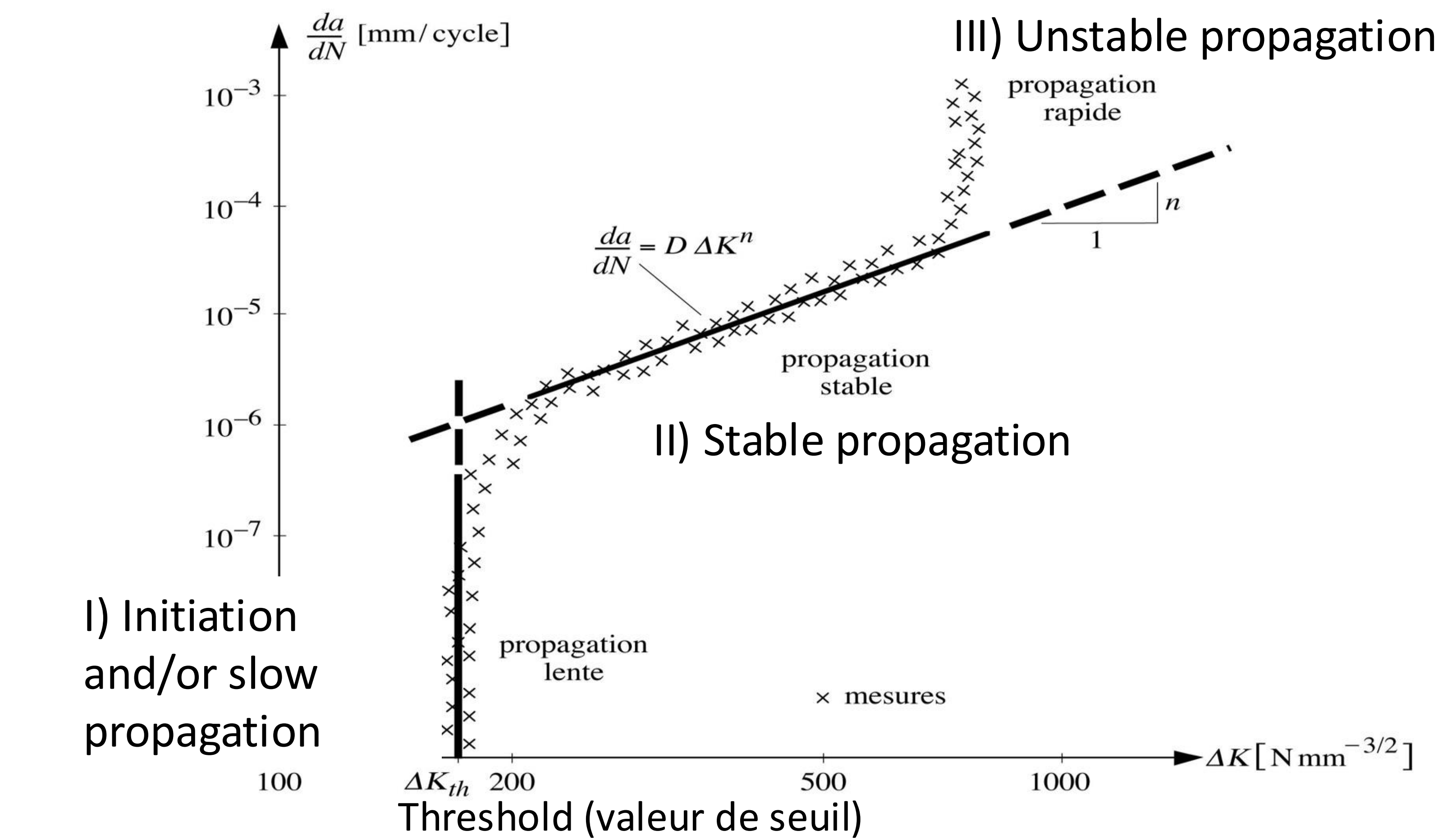


Fig. 13.10: Propagation law



Propagation laws

$$\frac{da}{dN} = D \cdot \Delta K^{n} \qquad \Delta K > \Delta K_{th} \qquad \text{(Stage II, Paris law)}$$

$$\frac{da}{dN} = 0 \qquad \Delta K \leq \Delta K_{th}$$

Other expression (Stages 1 and 2):

$$\frac{da}{dN} = D \cdot \left(\Delta K^{n} - \Delta K_{th}^{n} \right) \geq 0$$

Another expression, Forman (Stages 2 and 3):

$$\frac{da}{dN} = D_2 \cdot \frac{\Delta K^{n_2}}{K_c \cdot (1 - R) \cdot \Delta K} \geq 0$$

Fatigue service life, by integration (section 13.3.3)

$$\frac{da}{dN} = D \cdot \mathrm{D}K^n \qquad \mathrm{D}K > \mathrm{D}K_{th}$$

$$N = \int_{a_i}^{a_j} dN = \int_{a_i}^{a_j} \frac{1}{D \cdot \Delta K^n} da$$

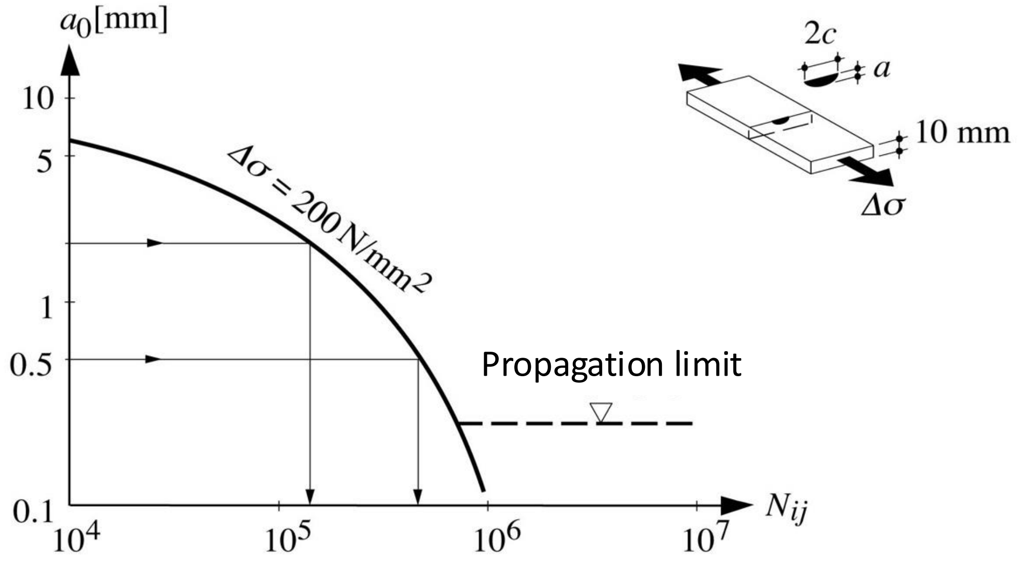
Formal integration possible for simple cases

$$N = \frac{1}{D \cdot \alpha \cdot \pi^{n/2} \cdot Y^n} \frac{1}{\Delta\sigma^n} \frac{1}{a_i^{\alpha}} \left[1 - \left(\frac{a_i}{a_j} \right)^{\alpha} \right]$$

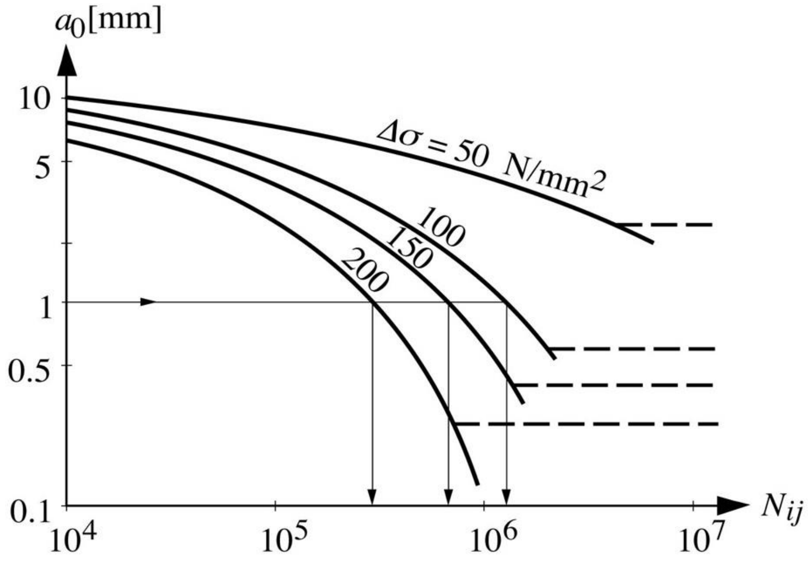
$$N = \int_{a_0}^{a_{cr}} dN = cte \cdot \Delta\sigma^{-n}$$

$$a = n/2 - 1$$

Fig. 13.16: Parameters influencing service life

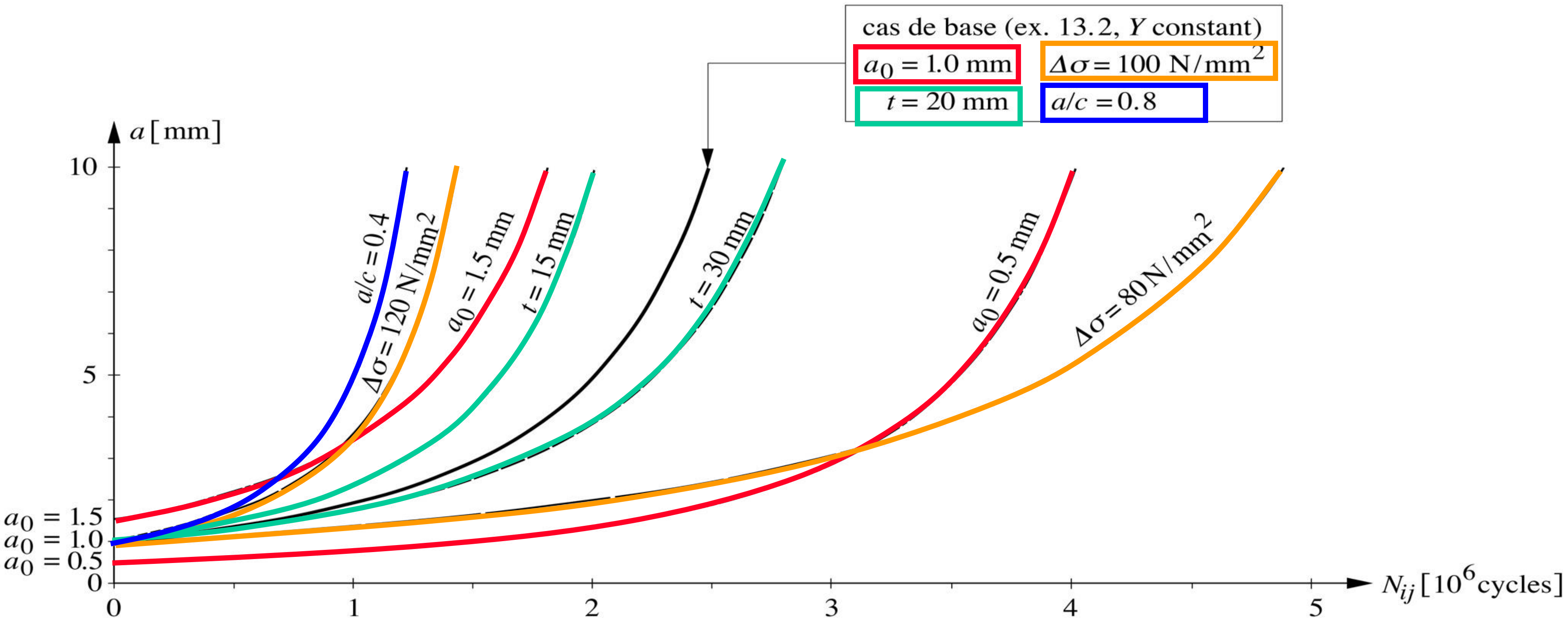


(a) Effect of the initial crack dimension a_0



(b) Effect of the stress difference $\Delta\sigma$

Fig. 13.17: Comparison between various N-a curves



Thickness effect in welded joints

- For Y = cte, $M_k = v \cdot (a/T)^w$ (valid for many welded joints) and $n \neq 2$

$$N = \frac{T^{nw}}{D \cdot \alpha' \cdot \pi^{n/2} \cdot v^n \cdot Y^n} \frac{1}{\Delta\sigma^n} \frac{1}{a_i^{\alpha'}} \left[1 - \left(\frac{a_i}{a_j} \right)^{\alpha'} \right]$$

$$\alpha' = 1 - n\left(w + \frac{1}{2}\right)$$

scale or thickness effect !

- Since n = 3 (= *m*, steel) and *w* = -0.33 to -0.10 (attachments & transversal welds, ...), then *N* decreases when *T* increases!
- In the codes, expressed as:

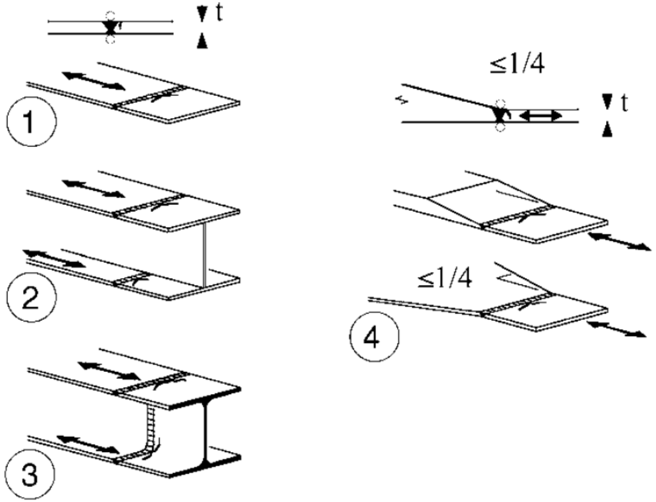
$$k_n = \left(\frac{N_T}{N_{Tref}}\right) \cong \left(\frac{T}{T_{ref}}\right)^{nw}$$

$$\left(\frac{\Delta\sigma_{nom,T}}{\Delta\sigma_{nom,Tref}}\right)^{-3} = \left(\frac{T}{T_{ref}}\right)^{-3\cdot w}$$

$$\boxed{\boldsymbol{k_s} = \frac{\Delta\sigma_{nom,T}}{\Delta\sigma_{nom,Tref}} = \left(\frac{T_{ref}}{T}\right)^{-w}}$$

Thickness effect in welded joints, extract EN 1993-1-9

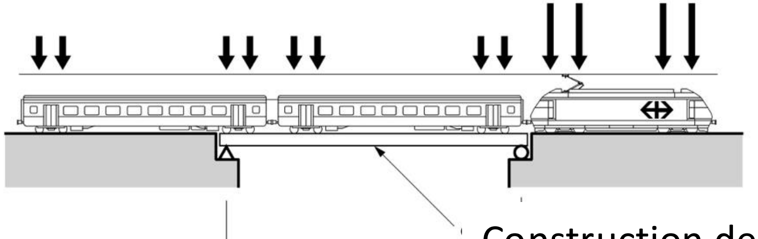
Table 8.3: Transverse butt welds

Detail category	Constructional detail		Description	Requirements
FAT $\Delta\sigma_C$ 112	Size effect for t > 25 mm: $k_s = (25/t)^{0,2}$		Without backing bar: 1) Transverse splices in plates and flats. 2) Flange and web splices in plate girders before assembly. 3) Full cross-section butt welds of rolled sections without cope holes. 4) Transverse splices in plates or flats tapered in width or in thickness, with a slope ≤ ¼.	- All welds ground flush to plate surface parallel to direction of the arrow. - Weld run-on and run-off pieces to be used and subsequently removed, plate edges to be ground flush in direction of stress. - Welded from both sides; checked by NDT. Detail 3): Applies only to joints of rolled sections, cut and rewelded.
			5) Transverse splices in plates or …	- The height of the weld convexity …

Size effect consideration: $\Delta\sigma_{C,red} = \boldsymbol{k_s} \cdot \Delta\sigma_C$

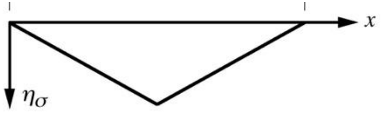
Structures under real loading: Variable amplitude loading

Loading model and statical system

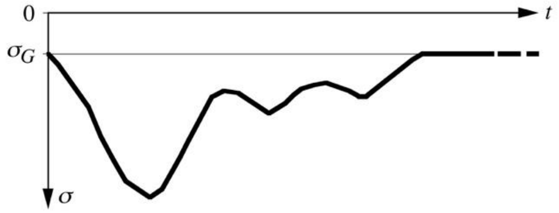


Construction detail

Influence line (stress in considered detail)



Historic of stresses



Structures under real loading: fatigue life, propagation stages I and II (but without initiation)

$$N = \int_{a_i}^{a_j} dN = \int_{a_i}^{a_j} \frac{1}{D \cdot \left(\Delta K^n - \Delta K_{th}^n \right)} da$$

Under variable amplitude (cycle-by-cycle propagation):

$$a_f = a_i + D \cdot \sum_{j=1}^{N} \left(\Delta K_j^n - \Delta K_{th}^n \right)$$

$$\Delta K_j = Y_j \cdot \Delta\sigma_j \cdot \sqrt{\pi a_j}$$

C-Mn steels, average values:

D_{moy} = 2.0·10^{-13} (mm/cycle)·(N/mm−3/2)^3 , n = 3

$\Delta K_{th,\text{moy,R = 0}}$ = 100 MPa√mm (for a stress ratio $R = 0 = \frac{\sigma_{min}}{\sigma_{max}}$)